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# Statistical Quality Technology

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# Statistical quality technology

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Enhancing industrial quality requires communication between many individuals each faced with unique responsibilities. All are part of a learning process requiring information-laden data and quantitative communication skills. Statisticians increase the speed of this learning process by emphasizing the simplest tools of statistics, most particularly graphics and elementary experimental designs.

## The role of statistics

The first key to solving a quality problem is to recognize that everyone from the executive office to the production floor is engaged in a learning process. A second key is to describe the quality problem in the highest level of language possible. Unfortunately, the specialized languages of the executive, engineer, foreman and worker do not facilitate ready communication, and parochialism often prohibits learning together. Fortunately, the arts of statistics were created to speed quantitative learning and communication.

It is instructive now to review events in Japan at the conclusion of World War II. In 1950 Mr Ichiro Ishikawa, president of the Japanese Union of Science and Engineering (JUSE), asked Dr W. Edwards Deming of the United States to speak to the industrial leadership of Japan on the subject of quality. One consequence of the Deming lectures was an extensive JUSE education programme that provided courses on statistical methods to tens of thousands of production foremen and engineers and, more to the point, to the executive officers of Japanese industry. Quoting Dr Deming, 'In Japan, statistical symbols and methods became a second language for everybody, including hourly workers.'

As part of this national exercise in statistical education the short text *Guide to quality control* was published, edited by Dr Kaoru Ishikawa, and later (1971) translated into English. The *Guide to quality control* places its initial emphasis on graphical aids to problem solving: check sheets for good number librarianship, histograms, cause-and-effect (fishbone) diagrams and Pareto charts. The text describes how to estimate the mean and standard deviation, to make simple $x$–$y$ plots, and how to use binomial probability paper to express information on percentages. Additional topics covered include simple sampling inspection schemes and quality control charts. Ishikawa's book kept the methodology simple and essentially graphical. The Japanese Union of Scientists and Engineers made sure that everyone, from executive to worker, got the message. The lesson to be drawn is clear.

## Quality control charts

The Shewhart (1931) or Dudding–Jennett (1942) average and range ($\bar{x}$ and $R$) control chart represents a high level of graphical exposition. The chart is a visual signal-to-noise ratio; each plotted point is compared with appropriate control limits based upon a measure of process variability. As causes are assigned to the unusual events identified by control charts, a more stable manufacturing process is established.

There are many variants of the $\bar{x}$ and $R$ control chart: the p chart for percentages, the c chart for counts, charts that use the estimate of the standard deviation in place of the range, charts for individual operations, running averages, medians and midranges. Acceptance control charts providing protection against both type I and type II errors were published by R. Freund (1957). Jackson (1959) provides examples of multivariate control charts that use Hotelling's $T^2$.

In the process industries, as opposed to piece and assembly industries, the Shewhart chart is often accompanied or replaced entirely by the cumulative sum (CUSUM) chart, a distinct British contribution to statistical quality control, see Page (1954), Barnard (1959), Kemp (1961) and Goldsmith & Whitfield (1961). A CUSUM chart is a sequential plot of the sum of the successive differences between observed values and a fixed target value and is used to monitor shifts in process mean away from target. The control limits for the CUSUM consist of a V-mask placed horizontally, $>$, a fixed distance in advance of the last plotted CUSUM point. CUSUM schemes are also available for monitoring one-sided shifts of mean from target and can be adapted to monitor percentage and count data (Johnson & Leone 1977) as well as variance (British Standards Institute 1980).

Whereas the Shewhart chart provides a visual impression of process 'noise' (random scatter), the CUSUM chart effectively smooths the data and provides a plot that is sensitive to small shifts in mean away from target. By studying the CUSUM time trace the quality control analyst can uncover clues as to both the time of origin and the size of the shift in mean. Lucas & Crosier (1982) recommend that an initial non-zero sum be used to enhance the performance of the CUSUM chart whenever a chart is restarted after a process correction has been made.

The exponentially weighted moving average (EWMA) chart is still another quality control charting procedure. The EWMA can be viewed two ways, as a linear statistic that places less and less importance on historical data, (a statistic with a 'forgetter'), or as a forecast function for a simple non-stationary process. The expression for the EWMA is

$$\hat{y}_{t+1} = \hat{y}_t + \lambda e_t = \hat{y}_t + \lambda(y_t - \hat{y}_t),$$

where

$\hat{y}_{t+1}$ is the predicted value (EWMA) at time $t+1$;
$\hat{y}_t$ is the predicted value (EWMA) at time $t$;
$y_t$ is the observed value at time $t$;
$e_t$ is the forecast error equal to $(y_t - \hat{y}_t)$;
$\lambda$ is the weighting parameter, $0 < \lambda \leqslant 1$.

Largely unrecognized is the ease with which the EWMA can be co-plotted simultaneously with observed values $y_t$ (see figure 1). A recent expository paper on the EWMA by Hunter (1986) gives detailed computations and examples.

The ability of the EWMA to provide a forecast offers the opportunity for real-time control. Further, the EWMA can be augmented by adding additional terms consisting of the sum and first difference of the errors $e_t$, i.e.

$$\hat{y}_{t+1} = \hat{y}_t + \lambda_1 e_t + \lambda_2 \sum e_t + \lambda_3 \nabla e_t,$$

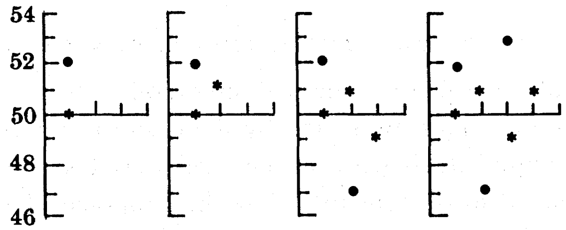



FIGURE 1. Plotting the EWMA. The EWMA is plotted as $*$, observations as $\bullet$. The EWMA $\hat{y}_{t+1} = \hat{y}_t + \lambda e_t$ is computed at time $t$ but plotted at time position $t+1$. To begin, let the first predicted value $\hat{y}_1 = 50$. The first observation $y_1 = 52$, thus $e_1 = 2.0$. Given $\lambda = 0.5$, then $\hat{y}_2 = 51$. The second observation $y_2 = 47$, $e_2 = -4.0$ and thus $\hat{y}_3 = 49$. Finally, $y_3 = 53$, $e_3 = 4.0$ and $\hat{y}_4 = 51$.

where $\nabla e_t$ indicates the first difference $e_t - e_{t-1}$. The three $\lambda$s are analogous to the proportional, integral and derivative terms, of the PID controller. The EWMA forms a communications bridge between classical statistical quality control and modern control engineering. The EWMA is, of course, only one of the many time-series models characterized by the Box & Jenkins (1976) autoregressive-integrated-moving-average (ARIMA) class of models.

## The modern arts of charts

Recent years have seem remarkable interest and advances in graphical methods as witnessed by the Sheffield conference in 1977, Fieller (1978), Cox (1978), and in the papers by Mahon (1977), Benigen & Robyn (1978) and Fienberg (1979). How to plot data badly has also received attention (Wainer 1984). The spirit of good graphics is captured in the book *The visual display of quantitative information* (Tufte 1983) and its research and development in the text *Exploratory data analysis* (Tukey 1977) and in the recent work of Cleveland & McGill (1987). Many of the new graphical techniques are applicable to the analysis of large masses of data and require the heavy use of the computer. But there are other simple graphics (earlier identified as 'metroglyphs' by Anderson (1960)) that everyone bearing responsibilities for quality should know and use: the box-and-whisker plots and the stem and leaf diagrams. For varied applications see Hoadley (1981), White & Schroeder (1987) and Iglewicz & Hoaglin (1987). A very recent metroglyph is the digidot plot: a stem-and-leaf plot (the digits) constructed simultaneously with a standard time-sequence plot of observations (the dots) as the data are sequentially observed (Hunter 1988). The digidot plot can completely replace the usual time sequence list of data as shown in figure 2.

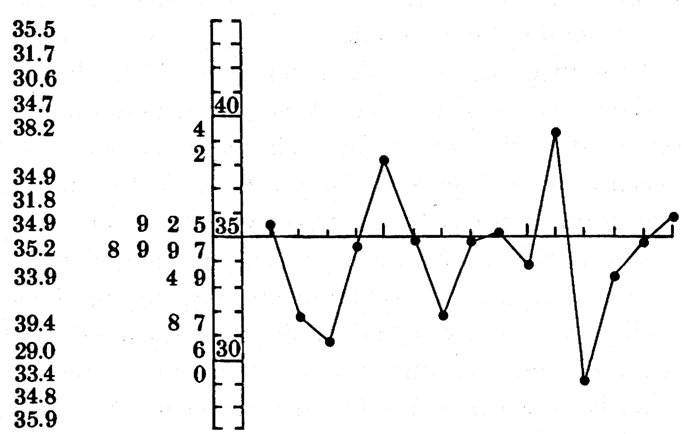



FIGURE 2. Digidot plot.

## Planning for information laden data

On observing a manufacturing process one will often notice that workers frequently make process changes, *ad hoc* modifications motivated by the desire to improve the process. Much would be gained if these manufacturing process changes were considered to be experiments. The challenge would then be to demonstrate that the proposed change was beneficial by designing a program to produce information descriptive of the consequences of the change.

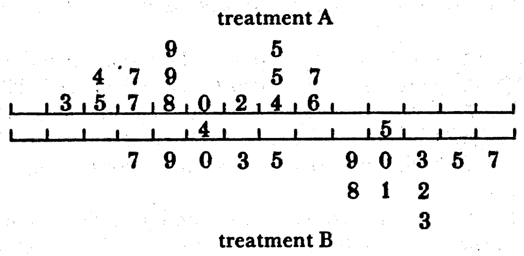
A philosophy and methodology for experimentation in a manufacturing environment was described by George Box (1957) in his paper 'Evolutionary operation: a method for increasing industrial productivity'. EVOP does not wait upon nor monitor the process in the hope of detecting signals, but instead forces the process to reply to carefully structured questions posed through the use of simple experimental designs. Most statistical methods used in industry are passive; they record and display the data as it arrives. EVOP is active.

Within a few years of its introduction, many papers appeared describing the successful use of EVOP programs, (Hunter & Kittrell 1966), and the expository text *Evolutionary operation* (Box & Draper 1969) was published. Still it must be acknowledged that EVOP has not become an active weapon in the armory of many of today's quality experts. One explanation may be the required transition from a passive to an active mind-set. A second explanation may be the mild complexity of the EVOP protocols, particularly the emphasis on multifactor experimentation.

There is nothing basically wrong with a 'one-factor-at-a-time' EVOP programme, the essential idea to plan a process change and then to repeat it, alternating between 'old' and 'new' operating conditions. The analysis of such programmes is simple and can be accomplished with a little arithmetic and graphics. One method is to plot the $t$ statistic constructed from the sequentially observed differences against boundaries derived by S. Rushton and G. A. Barnard as described in the excellent applied statistics text (Davies 1963). An alternative is to use the boundaries established by the National Bureau of Standards Tables for the sequential $t$ test (1951) and to plot the statistic $(\Sigma d_t)^2/\Sigma d_t^2$, where $d_t$ is the observed difference between paired observations representing the 'old' and 'new' process; see Armitage's (1975) text on sequential medical trials. A graphical method for comparing unpaired data is to construct a two-sample stem-and-leaf plot followed by a count of the exceedances, Tukey (1959), as shown in figure 3.

The eye provides a broad avenue of communication to the mind and in teaching analysts to read metroglyphs care must be taken to develop the capacity to distinguish between signal and noise. For example, on observing figure 3, one's eye might be attracted to the bimodality of both samples. Given the limited amount of data, this hint of bimodality is only that, a hint. Similar inference problems exist when analysing points plotted on probability papers; normal, log-normal, exponential, Weibull, etc. As an aid, the Lilliefors tests for normality and exponentiality have been converted onto special graph papers by Iman (1982).

Quality enhancement is not restricted to the production floor and we are led to consider the recent contributions of Taguchi (1986), most particularly as applied within the piece and assembly industries. Taguchi argues that to manufacture quality products it is not enough to reduce process variability, to screen out bad products or to fine-tune the production line. Equally important is the design of products robust to manufacturing variability and to the environmental shocks likely to be experienced by the customer. The creation of such products is enhanced by the use of experimental designs, particularly the fractional factorial designs. Not




| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | 3.8 | 4.4 | 4.2 | 4.5 | 3.7 | 3.5 | 3.7 | 3.3 | 4.6 | 4.0 | 3.9 | 4.7 | 3.4 | 3.9 | 4.5 |
| B | 4.0 | 4.3 | 5.5 | 5.0 | 4.1 | 5.3 | 5.2 | 5.7 | 4.3 | 4.9 | 4.8 | 3.7 | 5.1 | 5.3 | 4.5 |

FIGURE 3. Two-sample stem-and-leaf plot. Count the number of exceedances, the number of observations in one treatment less (larger) than the least (largest) in the second treatment. In this example there are twelve. If the number of exceedances is seven or more the two samples are statistically significantly different with $\alpha = 0.05$.

surprisingly, the data analysis methods proposed by Taguchi often use simple graphics, the more formal computations ordinarily associated with factorial designs analysis largely shunned. When comparing $k$ averages a good graphical presentation is provided by a plot of the averages along with their sliding reference distribution as explained in the text by Box *et al.* (1978).

## Industrial learning process

The successful executive or worker must actively seek information, not just data. New information, combined with prior knowledge, helps in the anticipation and/or the solution of quality problems. In any campaign to improve product quality or to enhance process efficiency an essential first step should be to display historical and current data in graphical form. If the acquired state of knowledge is inadequate then experimental programmes should be planned to provide information-laden data. Once again, graphical exposition of the experimental data is important. When quantitative information is to be transmitted a picture is worth a thousand numbers.

Statistical methods have always played a role in the creation of quality products and in the enhancement of productivity. Remarkably, it is not the theory of the statistics that contributes most, here but rather the application of simple statistical methods, many of which are graphical. The quality contribution of statistics might best be expressed by $\text{Qual}_{\text{stat}}$, where

$$\text{Qual}_{\text{stat}} = \Sigma \text{ (statistical tools) (frequency of use).}$$

The benefits of statistical quality technology are most rapidly advanced by encouraging the application of the simplest tools.

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